Computational Social Choice

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These introductory slides accompany Chapter 6 of the Book

Multiagent Systems (G. Weiss, ed.)

http://www.the-mas-book.info

More detailed slides can be obtained from the authors' homepages.

Motivation

- *What is "social choice theory"?*
  - How to aggregate possibly conflicting preferences into collective choices in a fair and satisfactory way?
    - **voting** (e.g., political, but also wikipedia, facebook, debian)
    - resource allocation, fair division (e.g., cake cutting)
    - coalition formation, matching (e.g., house allocation, college admission)
    - webpage ranking (e.g., search engine aggregators, pagerank algorithm)
    - collaborative filtering (e.g., amazon or ebay)
  - Origins: mathematics, economics, and political science
  - Essential ingredients
    - Autonomous agents (e.g., human or software agents)
    - A set of alternatives (usually finitely many)
    - Preferences over alternatives
    - Aggregation functions

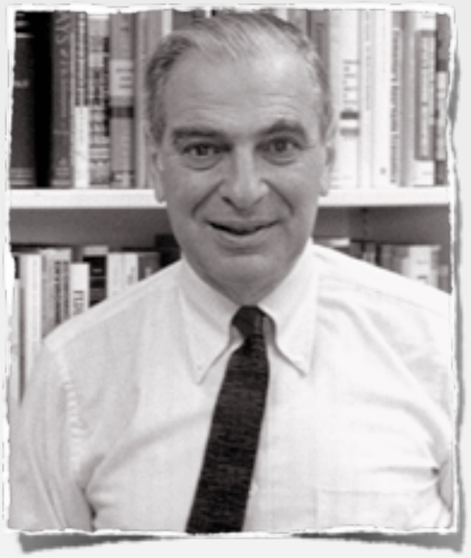
Key Questions

- What does it mean to make rational choices?
- Which formal properties should an aggregation function satisfy?
- Which of these properties can be satisfied simultaneously?
- How difficult is it to compute collective choices?
- Can voters benefit by lying about their preferences?

Recommended Books

- Introductory
  - H. Moulin: *Axioms of Cooperative Decision Making*. Cambridge University Press, 1988
  - W. Gärtner: *A Primer in Social Choice Theory*, Oxford University Press, 2009
  - M. Allingham: *Choice Theory - A very short introduction*. Oxford University Press, 2002
- Advanced
  - D. Austen-Smith and J. Banks: *Positive Political Theory I & II*, University of Michigan Press, 1999 & 2005
  - J. Laslier: *Tournament Solutions and Majority Voting*. Springer-Verlag, 1997
  - A. Taylor: *Social Choice and the Mathematics of Manipulation*, Cambridge University Press, 2005.

- Amartya Sen
  - Nobel prize 1998
- Kenneth J. Arrow
  - Arrow's impossibility theorem
  - Nobel prize 1972
- John George Kemeny
  - 1926-1992
  - BASIC programming language
- Charles Dodgson (Lewis Carroll)
  - 1832-1898
- Marie Jean Antoine Nicolas Caritat (Marquis de Condorcet)
  - 1743-1794


Plurality

- Why are there different voting rules?
    - What's wrong with plurality (the most widespread voting rule) where alternatives that are ranked first by most voters win?
    - Consider a *preference profile* with 21 voters, who rank four alternatives as in the table below.

3	**5**	**7**	**6**
a	*a*	*b*	*c*
b	*c*	*d*	*b*
c	*b*	*c*	*d*
d	*d*	*a*	*a*

-
    - Alternative *a* is the unique plurality winner despite the fact that
        - a majority of voters think *a* is the worst alternative,
        - *a* loses against *b*, *c*, and *d* in pairwise majority comparisons, and
        - if the preferences of all voters are reversed, *a* still wins.

5 Common Voting Rules

- Plurality *(most democratic countries, ubiquitous)*
  - Alternatives that are ranked first by most voters
- Borda *(Slovenia, academic institutions, Eurovision song contest)*
  - The most preferred alternative of each voter gets *k-1* points, the second most-preferred *k-2* points, etc. Alternatives with highest accumulated score win.
- Plurality with runoff *(France)*
  - Two alternatives that are ranked first by most voters face off in a majority runoff.
- Instant-runoff *(Australia, Ireland, Malta, Academy award)*
  - Alternatives that are ranked first by the lowest number of voters are deleted. Repeat until no more alternatives can be deleted.
- Sequential majority comparisons *(US congress)*
  - Alternatives that win a sequence of pairwise comparisons.

A Curious Preference Profile

(due to M. Balinski)

33%	16%	3%	8%	18%	22%
a	*b*	*c*	*c*	*d*	*e*
b	*d*	*d*	*e*	*e*	*c*
c	*c*	*b*	*b*	*c*	*b*
d	*e*	*a*	*d*	*b*	*d*
e	*a*	*e*	*a*	*a*	*a*

- Who will win according to the 5 common voting rules?
  - Plurality
  - Borda
  - Sequential majority comparisons (say, a,b,c,d,e)
  - Instant-runoff
  - Plurality with runoff

Desirable Properties (Axioms)

- Anonymity
  - The voting rule treats voters equally.
- Neutrality
  - The voting rule treats alternatives equally.
- Monotonicity
  - A chosen alternative will still be chosen when it rises in individual preference rankings (while leaving everything else unchanged)
- Pareto optimality
  - An alternative will not be chosen if there exists another alternative such that all voters prefer the latter to the former.

	Anonymity	Neutrality	Monotonicity	Pareto
Plurality	✓	✓	✓	✓
Borda	✓	✓	✓	✓
Plurality w/ runoff	✓	✓	-	✓
Instant-runoff	✓	✓	-	✓
SMC	✓	-	✓	-

6	5	4	2
a	*c*	*b*	*b*
b	*a*	*c*	*a*
c	*b*	*a*	*c*

6	5	4	2
a	*c*	*b*	*a*
b	*a*	*c*	*b*
c	*b*	*a*	*c*

Runoff rules fail monotonicity

Strategic Manipulation

- So far, we assumed that the *true* preferences of all voters are known.
- This is an unrealistic assumption because voters may be better off by misrepresenting their preferences.
- Plurality winner *a*
  - *b* wins if the last two voters vote for *b*, whom they prefer to *a*.

1	2	2	2
a	*a*	*b*	*c*
b	*c*	*d*	*b*
c	*b*	*c*	*d*
d	*d*	*a*	*a*

- How about Borda?
  - *a*'s score: 9, *b*'s score: 14, *c*'s score: 13, *d*'s score: 6
  - *c* wins if the voters in the second column, who prefer *c* to *b*, move *b* to the bottom.


Allan Gibbard

Gibbard-Satterthwaite Theorem


Mark A. Satterthwaite

- Why is manipulation undesirable?
  - Spending energy and resources on manipulative activities will be rewarded.
  - Manipulative skills are not spread evenly across the population.
  - Predictions or theoretical statements about voting rules become extremely difficult.
- Every reasonable voting rule is prone to manipulation whenever there are more than two alternatives.
  - Gibbard-Satterthwaite impossibility theorem (1973/75)
- Research in computational social choice has investigated the question of whether manipulation can be made computationally difficult.

Hardness of Manipulation

- Finding a beneficial manipulation for the following voting rules is NP-hard:
  - Second-order Copeland (Bartholdi, Tovey, and Trick; 1989)
  - Instant-runoff (Bartholdi and Orlin; 1991)
  - Nanson's rule (Narodytska et al.; 2011)
- Many more similar results for weighted voting and coalitional manipulation.
  - Key problem: NP-hardness is a worst-case measure
  - A string of recent results has cast doubt on this strand of research, culminating in work by Isaksson et al. (2010).
  - Essentially, they show that for every efficiently computable, neutral voting rule, a manipulable preference profile with a corresponding manipulation can easily be found.

Probabilistic Voting Rules

- Another idea to circumvent the Gibbard-Satterthwaite impossibility is to introduce randomization.
- Probabilistic voting rules yield probability distributions (so-called lotteries) over alternatives.
  - Random dictatorship: Pick a voter a random (*independently* of the voters' preferences) and choose his favorite alternative.
- Unfortunately, there is another far-reaching negative result.
- Whenever there are more than two alternatives, every non-manipulable, Pareto-optimal, probabilistic voting rule has to be a random dictatorship (Gibbard; 1977).

Strategic Abstention

- Consider the following preference profile and plurality with runoff.
  - Alternative a wins.
  - If two voters of the last column do not vote, c wins.
  - These voters prefer c to a.

4	**3**	**4**
a	c	b
b	a	c
c	b	a

- Voters in the last column are better off by abstaining, i.e., by not voting at all.
- Plurality and Borda are resistant to strategic abstention.
  - If winner changes from a to b by abstaining, the abstainer deducts strictly more points from a than from b.
- Most other voting rules suffer from strategic abstention.

Examples of Other Voting Rules

- Young's rule
  - If an alternative wins against every other alternative in pairwise majority comparisons, it is called a Condorcet winner.
  - Young's rule yields alternatives that can be made a Condorcet winner by removing as few voters as possible.
  - Computing Young winners is NP-hard!
- Approval voting
  - Rather than having complete preference rankings, voters only approve or disapprove of alternatives.
  - The alternative with the most approvals win.
- Range voting
  - Voters assign up to 100 points to each alternative.
  - Alternatives with maximal scores win.